Yet More Evidence for the Emptiness of Plurality

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1 Introduction

- Main Issue: What is the semantics nature of plural marking?

  1. Does it actually mark semantic plurality (the generation of groups and such) as suggested by Link (1983), Lasersohn (1995), and Chierchia (1998) among others?
  2. Is plural marking meaningless with respect to semantic plurality (as suggested by Sauerland, 2003; Sauerland, Anderssen, & Yatsushiro, 2005; Borer, 2005; and Krifka, 1995)?

- In this talk, I will present evidence for the second thesis. **Plural marking DOES NOT mark semantic plurality!**
- EVIDENCE: A mixture of new and old problems that disappear once the plural morpheme is not interpreted as a function that creates pluralities (groups) from singularities (individuals).
- THREE PROBLEMS (to be reviewed and then solved):

  1. The Problem of too many pounds.
  2. The paradox of grams.
  3. Plural agreement for numbers less than one.

- **PSEUDO-PARTITIVES and MEASURE NOUNS**
  - The nature of these problems stems from measure nouns such as *pounds* and *grams* in pseudo-partitive constructions like *pound of potato* and *grams of apples.* [1]
  - Note I assume that the plural marking on measure nouns is no different from the plural marking on other nouns. In this way, my approach to measure nouns has a different emphasis than Schwarzschild (2002), where plural marking remains unanalyzed.
- THE PROBLEMS DISAPPEAR when...

  1. all nouns (without plural marking) are treated as inherently plural;

[1] The term pseudo-partitives comes from Selkirk, 1977.

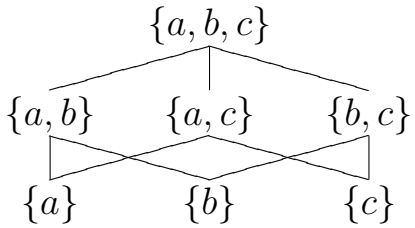
2. the plural morpheme is treated as an identity function;
3. numerals and the singular morpheme are treated as restrictors that shrink the denotation of the nominal they modify.

2 An interpretation for Plurality

- Traditional treatment of the plural morpheme hypothesizes that it creates pluralities from a set of singularities.
- **Example:** Link (1983) proposed that the plural morpheme should be interpreted as a function that forms a set of pluralities from a set of singularities. A definition of a function similar to Link's is given in (1). [2]

(1) a. $[\![PLURAL]\!] = PL$
b. $PL(X) = \{Y : Y \subseteq X \ \& \ Y \neq \emptyset\}$[3]

- The groups in the denotation of the plural are ordered with respect to the subgroup (or subset) relation. Given a singular denotation of $\{a, b, c\}$, the plural function would derive the following groups and the following ordering of groups (see 2).

(2)


- This theory of plurality, where the plural denotation is derived from the singular denotation, encounters 3 problems.

3 Three Problems

3.1 The Problem of Too Many Pounds

- There are many different ways in which a substance can be measured and furthermore singular NPs seem to be able to make reference to different types of measurements.
- **Example**: Consider a context where there is a two pound lump of mashed potato sitting in a bowl. For argument's sake, suppose that some grey food dye was spilled, staining the center of the lump of potato. (see figure 1)
- In this scenario, the following sentences can be truthfully uttered.

[2]Note I have simplified this function slightly for exposition purposes. I have added a set notation and I have allowed the plural morpheme to include atoms in the plural denotation, two features that are absent from Link's original proposal.

[3]I could have used the notation *POW* since the plural function is equivalent to the power set function subtracting the empty set.

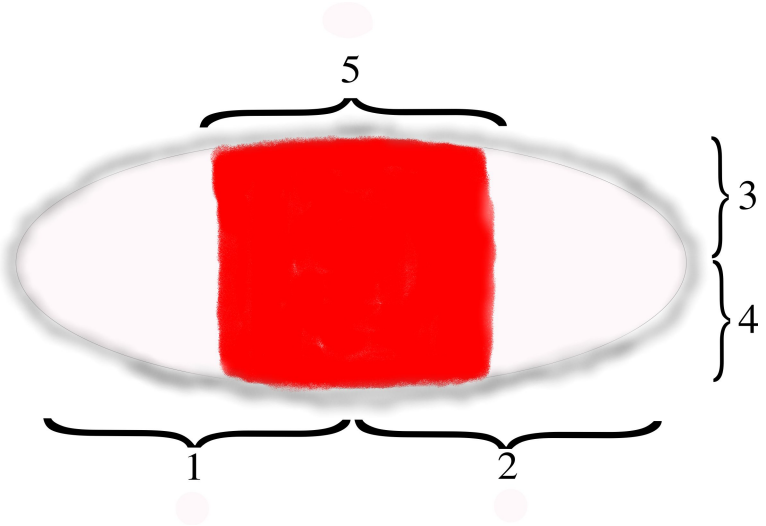


Figure 1: A graphical representation of the two pound lump of mashed potato sitting on the table. The vertical strip in the center represents the spilt food dye. The bracket represent the different places pointed to in the order indicated by the numbering.

(3) a. This pound of potato is partially grey. (pointing to left-half, 1)
b. That pound of potato is also partially grey. (pointing to right-half, 2)
c. Furthermore, this other pound of potato is partially grey. (pointing to top, 3)
d. Also, that other pound of potato is partially grey. (pointing to bottom-half, 4)
e. However, this pound of potato is completely grey. (pointing to center, 5)

- For convenience lets label the different pounds of potato with the numbers 1 through 5. The felicity of the sentences in (3) suggest that all the possible pounds from 1 to 5 are members of the singular denotation.
- **UNDESIRED GROUPS:** if all these different pounds are members of the singular denotation, then the plural function will form lots of different groups. Specifically...

1. Since 1 and 2 are members of the singular denotation, the group $\{1, 2\}$ must be a member of the plural denotation ($\{1, 2\}$ is a subset of $\{1, 2, 3, 4, 5\}$).
2. Since 3 and 4 are members of the singular denotation, the group $\{3, 4\}$ is also a member of the plural denotation ($\{3, 4\}$ is a subset of $\{1, 2, 3, 4, 5\}$).
3. Since 1, 2 and 5 are members of the singular denotation, the group $\{1, 2, 5\}$ is a member of the plural denotation.

Hence one would expect the following (partial) denotations for *pounds of potato*, *two pounds of potato*, and *three pounds of potato*.

(4) a. $[\![\textit{pounds of potato}]\!] = \{..., \{1, 2\}, \{3, 4\}..., \{1, 2, 5\}...\}$

b. $[\![\textit{two pounds of potato}]\!] = \{..., \{1, 2\}, \{3, 4\}...\}$
c. $[\![\textit{three pounds of potato}]\!] = \{..., \{1, 2, 5\}...\}$

THE PROBLEM: The problem with the denotations in (4) is two fold.

1. They predict that it would be possible to point at the two pound lump of potato on the table and talk about and refer to it as *three pounds of potato*
   – there is a group, namely $\{1, 2, 5\}$, that has a cardinality of three.

   In other words, the sentence in (5) should be coherent in the context specified above.

   (5) At least three pounds of potato are on the table.

2. They predict that one can not use the phrase *the two pounds of potato* to refer to the two pound lump of potato on the table.
   – Given the denotation of *two pounds of potato*, the sentence in (6) should be unacceptable. The definite determiner requires there to be a unique two pound group. However, as shown in (4) there is no unique two pound group.[4]

   (6) I poured grey dye on the two pounds of potato.

3.2 The Paradox of Grams

- With the knowledge that two pounds is roughly equivalent to 900 grams, the sentences in (7) have similar truth conditions.

  (7) a. I put the two pounds of apples on the table.
  b. I put the 900 grams of apples on the table.

- **Problem?**, consider one possible representation of the singular NP *gram of apples*:

  (8) $[\![\text{gram of apples}]\!] = \{x : x \in [\![\textit{apples}]\!] \ \& \ \mu_g(x) = 1\}$

  The plural function when applied to (8) would create the plural denotation. However, since no apple weighs only one gram (an apple weighs about 150 grams), this singular denotation would be empty. Thus the plural denotation would be empty, contrary to the evidence given in (7).

- Consider another possible representation of the singular NP *gram of apples*, where $\preceq$ is a *material-part-of* relation:

  (9) $[\![\text{gram of apples}]\!] = \{x : x \preceq \iota([\![\textit{apples}]\!]) \ \& \ \mu_g(x) = 1\}$

[4] Amendments could be made to the theories to deal with one of the problems by prohibiting groups that contain two members that material over-lap with one another. This amendment could be made in the plural function itself by specifying that group formation prohibits material over-lap. However, the problem of uniqueness still remains.

The denotation in (9) is not empty. It contains all the apple-stuff that weighs one gram. The measure noun grinds up the apples and then quantifies over the bits.

However, the denotation in (9) predicts that the sentences in (10) should be acceptable and interpretable in a context where an apple weighs more than 150 grams. Yet, the sentences are anomalous.

(10) a. ? Give me 50 grams of apples.
b. ?? Give me one gram of apples.
c. ?? 50 grams of apples are rotten.

The sentences in (10) imply that apples weigh less than 50 grams or less than one gram.

3.3 Plurals for Quantities Less than One

- Numbers less than one lead to plural agreement.

(11) a. I bought 0.725 grams of saffron.
b. I paid a dollar fifty for the 0.725 grams of saffron sitting on the table.
c. This recipe calls for 0.75 kilograms of chicken.

- Note, this fact led Krifka (1995) to remark that "the selection of singular or plural forms seems to be a purely syntactic matter."

4 One Solution

- **Nature of the problem**
  - In creating pluralities from a set of singularities, the plural operator creates **undesired/unattested** groups. (Groups that are two big in the problem of *too many pound* and groups that are too small in the problem of *the paradox of grams*.
  - In creating pluralities from a set of singularities, one does not create entities that are less than one unit.
- **Nature of the solution**: Eliminate the pluralizing function!
  - Interpret the underlying noun, prior to singular or plural marking, as inherently plural (the denotation would contain groups as well as singularities).
  - Interpret the plural morpheme as an identity function.
  - Interpret the singular morpheme as a function that restricts the noun to singularities.
- Eliminating the pluralizing function solves all three problems.

4.1 Details of the solution

- One possible tact would be to have a degree variable serving as the argument to some measure function in the noun and also to have this variable explicitly bound by an abstraction operator (a lambda-operator). See Krifka (1995) for such a proposal.

- I will not take this tact here. (Such a tact would require hidden operators in order to have the nouns combine with determiners such as *the* and adjectives such as *blue*. Such hidden operators create difficulties, especially given phrases like *two blue cats*.)

- **THE SOLUTION:** All nouns have a hidden variable that serves as an argument to some measure function. This variable is existentially bound. To manipulate this variable, I will exploit some of the properties of **Dynamic Predicate Logic (DPL)**. (Thanks to Chierchia for suggesting this possibility to me.)

- **Main Aspect of DPL to Exploit**: Due to the way in which DPL treats existential quantifiers and conjunction (as functions from variable assignments to variable assignments), the formula schema in (12a) is truth-functionally equivalent to the schema in (12b).

  (12) a. $[(\exists n \Phi) \& \Psi]$
  b. $\exists n (\Phi \& \Psi)$

- In effect, the existential quantifier seems to be able to bind a variable outside of its syntactic scope when it appears as the first member of a conjunct. (This is a crude and inaccurate way of describing the system, however it does get the point across.)

1. **Interpretation of the Root Noun**: Given DPL's treatment of existential quantification, consider the following possible interpretation for *pound of potato*. (I will assume that $[\![$ *potato* $]\!]$ denotes all the mass of *stuff* that counts as potato.)

   (13) $[\![$ *pound of potato* $]\!] = \lambda x \, \exists n (x \in [\![potato]\!] \, \& \, \mu_{lbs}(x) = n)$,
   where μ_{lbs} is a function from entities to the measurement of those entities in terms of pounds.

   For simplicity I will treat the noun *pound* as forming a constituent with *of potato*. Number morphology will combine with this entire phrase. (Note, nothing crucial depends on this treatment.)

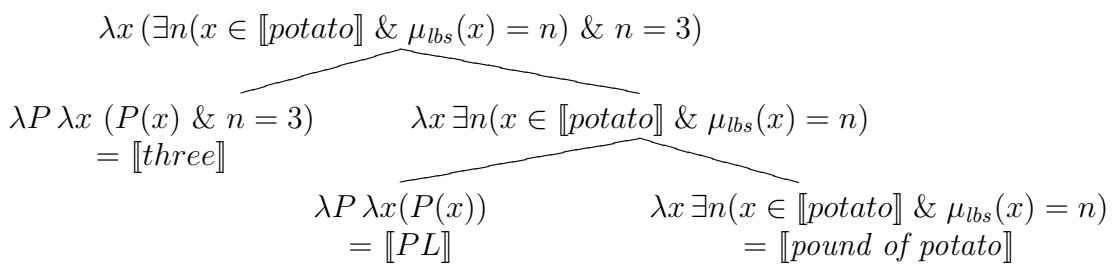
2. **Interpretation of the Plural Morpheme (PL)**: The plural morpheme will simply pass-up the value of the root noun.

   (14) $[\![PL]\!] = \lambda P \, \lambda x (P(x))$

3. **Interpretation of the Singular Morpheme (SG) and Numerals**: The singular morpheme and the numerals simply manipulate/restrict the possible values for the variable n within the root noun.

(15) a. $[\![two]\!] = \lambda P\, \lambda x(P(x)\ \&\ n = 2)$
b. $[\![three]\!] = \lambda P\, \lambda x(P(x)\ \&\ n = 3)$
c. $[\![SG]\!] = \lambda P\, \lambda x(P(x)\ \&\ n = 1)$

4. **Putting things together:** The interpretation of *three pounds of potato*:

(16)


Equivalence due to DPL

[[*three pounds of potato*]] IS EQUIVALENT TO $\lambda x\, \exists n(\ (x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\ \&\ n = 3)$

4.2 Solving the Three Problems

1. **The problem of Too Many Pounds - SOLVED!**

- If there is only a two pound lump of potato on the table, then the topmost member in the denotation of $[\![potato]\!]$ only weighs two pounds. There are no members that weigh three pounds and there is only one member that weighs two pounds.
- Thus the denotation of [[*two pounds of potato*]] is

(17) $\lambda x\, (\exists n(x \in [\![potato]\!]\ \&\ \mu_{lbs}(x) = n)\ \&\ n = 2).$

The characterizing set[5] of this function is the set of entities that are member of the denotation of $[\![potato]\!]$ and weigh two pounds. There is only one such member (the whole lump)! **Hence, the definite determiner *the* can apply to this set and still be defined.**

- [[*the two pounds of potato*]] is the unique potato mass that weighs two pounds. There is no conflict to uniqueness.

2. **The Paradox of Grams - SOLVED!**

- The same type of interpretation is given to *grams of apples* as *pound of potato*, the only difference is in terms of the output of the measure function and the count noun complement.

(18) [[*grams of apple*]] $= \lambda x\, \exists n(x \in [\![apples]\!]\ \&\ \mu_g(x) = n)$

- For now, let's assume that the denotation of $[\![apple]\!]$ is the set of all apples and apple-groups.

[5] In DLP, the characterizing set of a function P, can be give as $\{x : P(x)$ is true with respect to the current variable assignment $i\}$. A formula Φ is true with respect to a variable assignment i iff $\exists h : \langle i, h\rangle \in P(x)$.

- For x to be in the characterizing set of $\lambda x\, \exists n(x \in [\![apples]\!] \,\&\, \mu_g(x) = n)$, it has to be an apple or apple group (that has some weight). Since apples weigh more than 150 grams, there are no entities in this set that weigh under 150 grams.
- If a numeral such as 30 restricts the characterizing set to things that weigh 30 grams, the result will be an empty set. No apple (or apple group) weighs 30 grams.
- If a numeral such as 900 restricts the characterizing set to things that weigh 900 grams, the result will not necessarily be empty. It is possible to have an apple-group that weighs 900 grams.

- **Since groups are not created by the plural morpheme from singulars, there is no need to have a singular denotation in order to have a denotation for *900 grams of apples*.**

3. **Plurals for Quantities Less than One - Explained!**

(a) PART I: Numbers less than one can combine with plural NPs.

- The plural NP *grams of saffron* would have the following interpretation:

(19) $[\![\ grams\ of\ saffron\ ]\!] = \lambda x\, \exists n(x \in [\![saffron]\!] \,\&\, \mu_g(x) = n)$

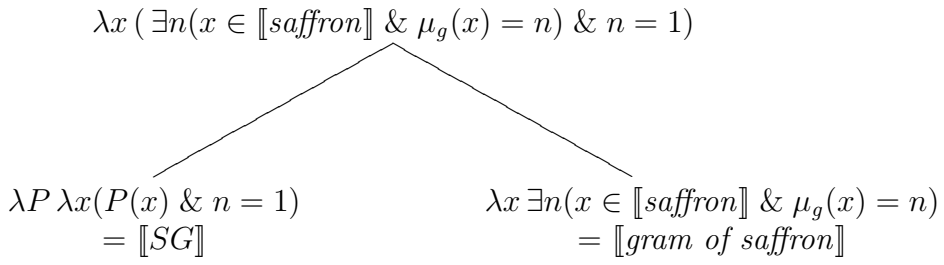
- Since n could be any number, the denotation of *grams of saffron* can contain x's that weigh less than 1.
- A numeral such as 0.75, in this system, would restrict the plural denotation to all the entities that weigh 0.75 grams.

- **Hence the characterizing set of** 0.75 ***grams of saffron*** **would contain all the saffron masses that weigh** 0.75 **grams.**

(b) PART II : Singulars cannot combine with numbers less than one.

- The singular NP *gram of saffron* would have the following interpretation.

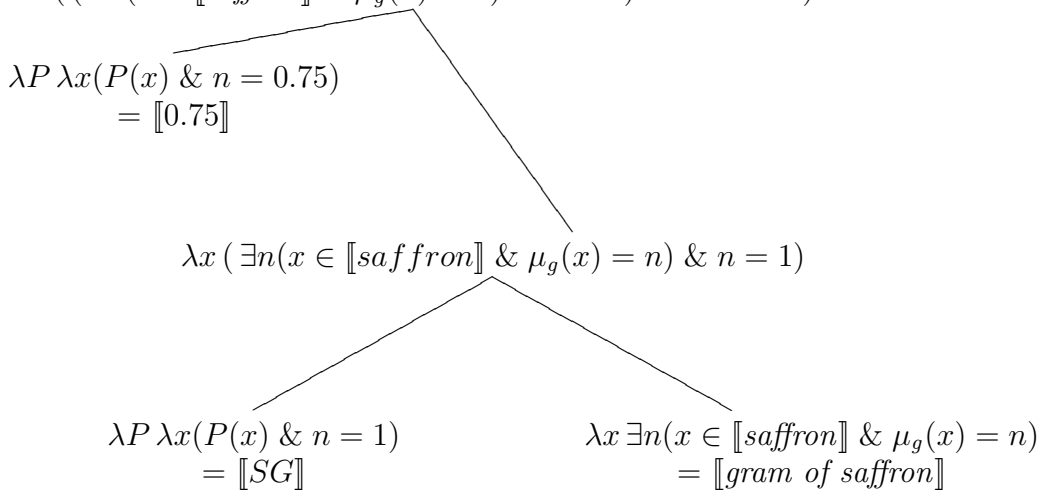
(20)



- The characterizing set of this interpretation would be the set of all masses of saffron that measured exactly one gram. There are no masses that weigh less than one.

- Restricting singular NPs to entities that measure 0.75 would be trivially empty. The function contains a contradiction.

(21) $\lambda x\,(\,(\,\exists n(x \in [\![\textit{saffron}]\!]\ \&\ \mu_g(x) = n)\ \&\ \mathbf{n = 1})\ \&\ \mathbf{n = 0.75})$



5 Implication and Consequences

5.1 Extending the analysis to all nouns

- This analysis can easily be extended to any count noun, such as *dog* for instance. It is possible that such a noun contains two parts:

  1. A basic concept that tell you what counts as a dog or a dog group. Let's label this concept DOG.
  2. A measure function that measures individuals and groups in terms of their atoms. Let's label this function AT.

- An existentially bound variable n could serve as the output of AT in regular count nouns. Thus *dog* would have the following interpretation...

(22) $[\![\ \textit{dog}\ ]\!] = \lambda x\,\exists n(DOG(x)\ \&\ AT(x) = n)$

- This noun would combine with numerals and the singular and plural morphemes in exactly the same way as *pound of potato*.

- **Hence, the plural and singular marking on measure nouns is treated the same as the plural and singular marking on regular count nouns!**

5.2 Revisiting the count/mass distinction

- If mass nouns do not contain any (existentially) bound variable n that relates to a measure function, then this would explain why numerals cannot combine with mass nouns. (This type

of explanation goes back to suggestions by Krifka, 1995, and has recently been revived by Borer ,2005.)

- Note, in the current system, such an explanation would allow for a mass noun to have the exact same kind of denotation as a count noun (namely an atomic denotation), yet the mass noun would not be able to combine with a numeral.
- Such mass nouns exist, namely *furniture*, *equipment*, *footwear*, *cutlery*, etc.
- The "hidden variable" analysis of plurality (advocated in this talk) explains how this fake mass nouns are formally different from count nouns but yet have identical types of denotations.

6 Conclusion

- I have shown that the plural morpheme, when interpreted as a pluralizing function, creates three problems.
  1. The problem of too many pounds.
  2. The paradox of grams.
  3. Plural agreement for numbers less than one.
- All three problem disappear if plural morphology is not treated as a pluralizing function.

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